ТЕХНИЧЕСКИЕ АСПЕКТЫ СТРОИТЕЛЬСТВА / TECHNICAL ASPECTS OF THE CONSTRUCTION

DOI: https://doi.org/10.18454/mca.2017.07.9
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COMPARATIVE ANALYSIS OF THE METHODS FOR EVALUATING THE EFFECTIVE LENGTH OF COLUMNS

Abstract

This article looks into the effective length of columns using different methods. The codes in use in this article are those from the AISC (American Institute of Steel Construction). And that of AS 4100 (Australian Steel code). A conclusion was drawn after investigating a frame using three different methods. Solved Exercise 6 (LeMessurier Method) was investigated using same frame but different dimension. Further analysis and investigation will be done using Java codes to analyze the frames.
Keywords: Effective Length factors of Column.

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СРАВНИТЕЛЬНЫЙ АНАЛИЗ МЕТОДОВ ОЦЕНКИ ЭФФЕКТИВНОЙ ДЛИНЫ СТОЛБЦОВ

Аннотация

В данной статье рассматривается метод оценки эффективной длины столбцов с использованием разных приемов. В статье используются коды AISC (Американского института стальных конструкций) и AS 4100 (Австралийский код стали). После исследования фрейма с использованием трех разных методов были сделаны соответствующие выводы. Упражнение 6 (метод Ле Мессурье) было решено с помощью того же фрейма, но другого измерения. Дальнейший анализ и исследование будут проводиться с использованием Java-кодов для анализа фреймов.
Ключевые слова: факторы, влияющие на эффективную длину столбца.
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Many codes of practice rely on the effective length method to investigate the stability of frames. The effective length method asses to the buckling capacity of a member in a structural system to be calculated by considering an equivalent pin ended column in Euler buckling [5].

The effective length concept is a method of estimating the interaction effect of the total frame on a compression element being considered.

The effective length uses the factor K equates the strength of a framed compression element to the actual column length L to an equivalent pin-ended column of length KL. The effective length factor K can be explained to be a factor which, when multiplied by the actual unbraced column length L of an end-restrained compression member, will produce an equivalent pin-ended member whose buckling strength is same as that of the original end-restrained member.

The K factor is responsible for the effect of restraint conditions on the behavior of the column and KL stands as the length over which the column buckles [8]. The effective length factor K can be determined using the methods below:

- Tabulated factors for stand-alone columns with well-defined support conditions.
- Alignment charts for columns in a rigid framed structure.
- Lui's method.
- LeMessurier Method.

The value of the effective length factor depends on the translational and rotational restraints at the ends of each member. Figure 1.1 gives both theoretical and recommended values of K for columns with idealized conditions of end restraint.

TABULATED FACTORS

AISC 360 Table C-A-7.1[1] specifies effective length factors for well-defined, standard conditions of restraint and these are illustrated in Fig. 1 for sway and non-sway columns. The values of K shown in figure 1 recommended for design are a little higher than their theoretical equivalents. This is as a result of the fact that joint fixity is seldom fully realized (i.e. the ends will always have some flexibility in them). The values for theoretical end conditions are indicated and also recommended values that allow for practical site conditions. These values may only be used in simple cases when the tabulated end conditions are approximated in practice.

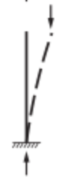
Braced frame				Sway frame			
End restraints	Ideal K	Practical K	Shape	End restraints	Ideal K	Practical K	Shape
Fixed at both ends	0.5	0.65		Fixed at one end with the other end fixed in direction but not held in position	1.0	1.2	
Fixed at one end, pinned at the other end	0.7	0.8		Pinned at one end with the other end fixed in direction but not held in position	2.0	2.0	
Pinned at both ends	1.0	1.0		Fixed at one end with the other end free	2.0	2.1	
Symbols for end restraint conditions	= Rotation fixed, translation fixe; = Rotation free, translation fixe; = Rotation fixed, translation fixed; = Rotation free, translation free						

Fig. 1 - Effective length factors for sway and non-sway columns

K Values for Braced Frames

Braced frames may be analyzed and designed as vertical cantilevered pin-connected truss systems, not considering any secondary moments in accordance with AISC 360 Commentary Sec. A-7.2 [1]. The effective length factor for components of the frame is taken as 1.0.

Solved Exercise 1. Braced Frame Effective Length Factors

For the braced frame shown in Fig. 2, determine the effective length factors of the columns. The girder may be considered infinitely rigid and the columns are adequately braced in the transverse direction.

The effective length factors may be obtained from Fig. 1.1. For column ab which is fixed at both ends K = 0.65. For column cd is also fixed at both ends K = 0.65.

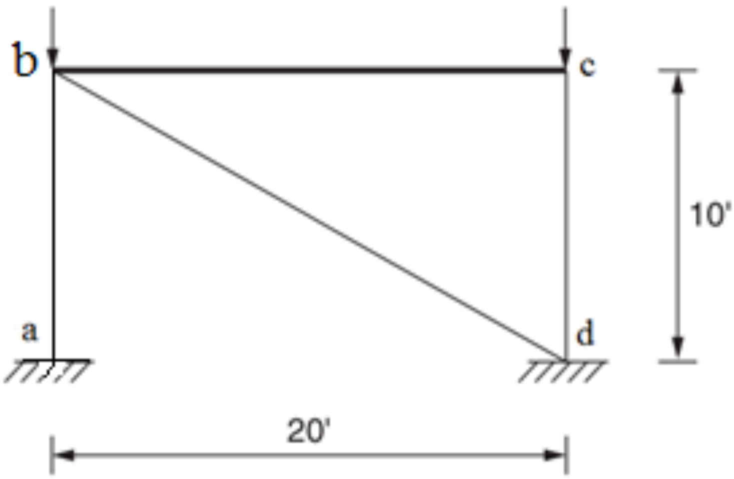


Fig. 2 - Details for Solved Exercise 1

K Values for Sway Frames

Effective length factors of sway columns are similarly derived. Generally, the effective length factor exceeds 1.0 except for frames with high structural stiffness. For these frames, the side-sway amplification factor is

$$\mathbf{B_2 = \Delta_{2nd}/\Delta_{1st}} \tag{1}$$

Where; Δ_{2nd} is second-order drift and Δ_{1st} is first-order drift.

When $B_2 \leq 1.1$, AISC 360 Sec. A-7.2 [1] permits the use of an effective length factor of K = 1.0. Leaning columns are columns that provide little or nothing to the sway stiffness of a story or the defiance to lateral loads. These columns may be designed as pin-ended, with an effective length factor K = 1.0, in line with AISC 360 Sec. A-7.2 [1]. However, all other columns in the story must be designed to support the destabilizing P-Δ moments developed from the loads on the leaning columns.

Solved Exercise 2. Sway Frame Effective Length Factors

For the sway frame shown in Fig. 3, determine the effective length factors of the columns. The girder may be considered infinitely rigid and the columns are adequately braced in the transverse direction. For column ab which is fixed at both ends K = 1.2. For column cd which is fixed at both ends K = 1.2.

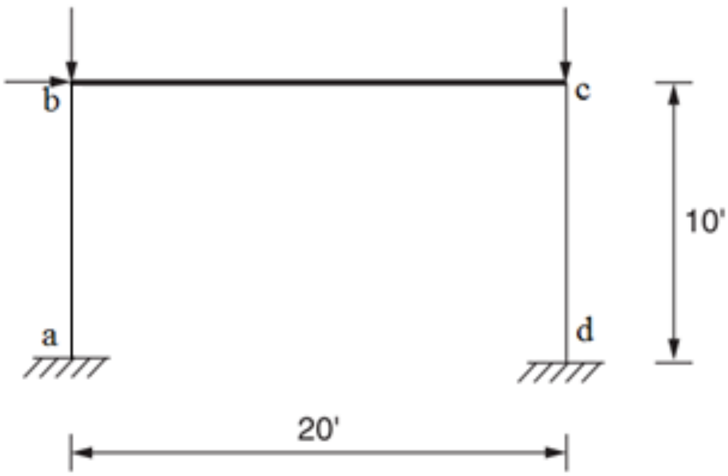


Fig. 3 - Details for Solved Exercise 2

ALIGNMENT CHARTS

The effective length factor of columns in a frame with rigid joints may be determined based on the restraint provided at every end of the column. The alignment charts shown in figure 4 derived from Fig.C-A-7.1 and C-A-7.2 of AISC [1]. To make full use of the alignment chart, the stiffness ratio at the two ends of the column under consideration must be determined and this is defined by equation 2.

$$G = \Sigma(E_c I_c/L_c)/\Sigma(E_g I_g/L_g) \tag{2}$$

Where;

$\Sigma(E_c I_c/L_c)$ - sum of the EI/L values for all columns meeting at the joint and

$\Sigma(E_g I_g/L_g)$ - sum of the EI/L values for all girders meeting at the joint.

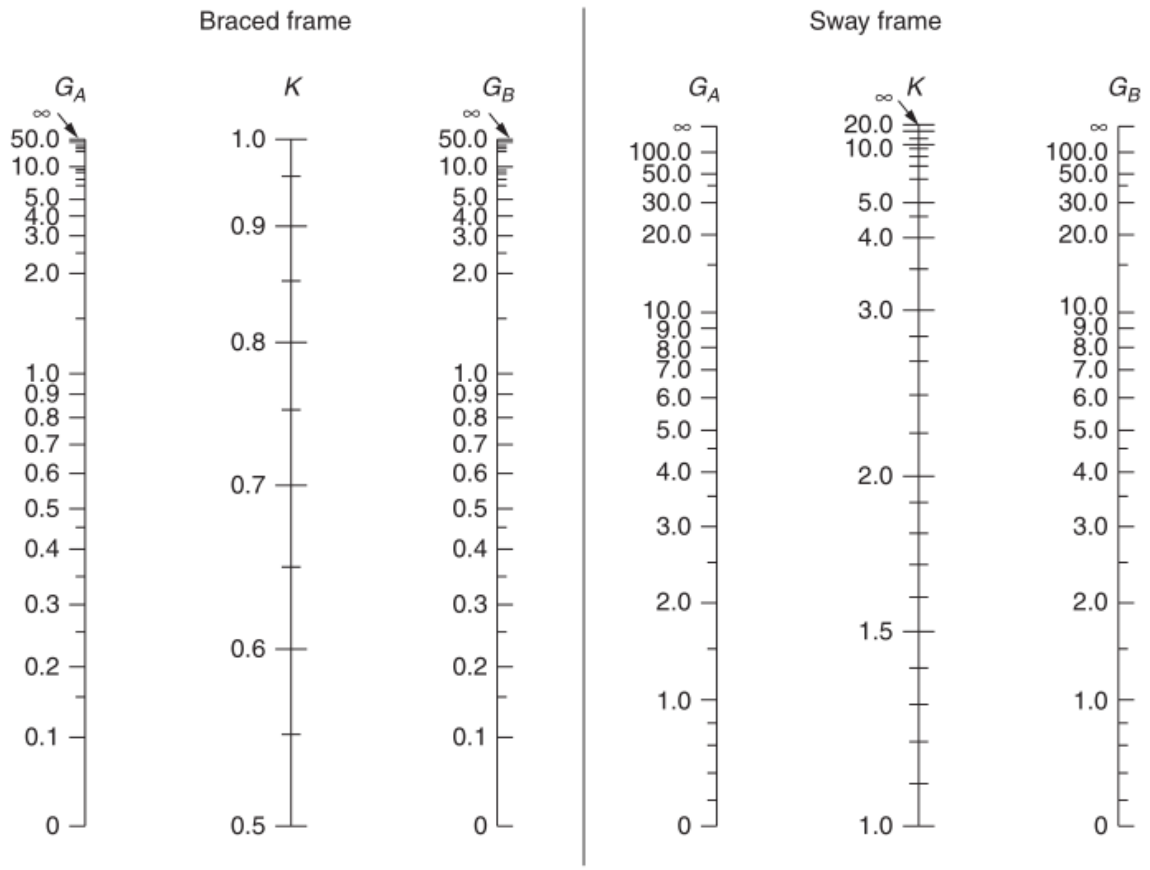


Fig. 4 - Alignment chart for effective length

Using the stiffness ratio G_A at one end of a column and the stiffness ratio G_B at the other end of the column, the alignment chart is entered and the effective length factor is obtained.

For a column with a pinned base, the stiffness ratio is theoretically infinity and a practical value of G = 10, is recommended by AISC 360 Commentary Sec. A-7.2[1]. For a column with a fixed base, the stiffness ratio is theoretically zero and AISC 360 Commentary Sec. A-7.2 recommends a practical value of G = 1.0.

Alignment Chart for Braced Frame

For a braced frame, with rigid joints, the girders are assumed bent in single curvature and the alignment chart is based on a stiffness value for the girders of *2EI/L*. If the far end of a girder is pinned, its stiffness is *3EI/L*, as determined in Part 2, Sec 7.4 Structural Analysis in Theory and Practice [2]. Hence, the calculated (E_gI_g/L_g) value is multiplied by 1.5 before determining the value of $\Sigma(E_gI_g/L_g)$ and entering the chart. If the far end of a girder is fixed, its stiffness is 4EI/L, as determined in Part 2, Sec 7.4 Structural Analysis in Theory and Practice [2]. Hence, the calculated (E_gI_g/L_g) value is multiplied by 2.0 before determining the value of $\Sigma(E_gI_g/L_g)$ and entering the chart.

Solved Exercise 3.Braced Frame Effective Length Factors by Alignment Chart

For the braced frame shown in Fig. 5, determine the effective length factor of column 21. The braces may be considered fixed at each end and the columns are adequately braced in the transverse direction. For the fixed connection at joint 1, AISC 360 Commentary Sec. A-7.2 [1] recommends a practical value of G_1 = 1.0

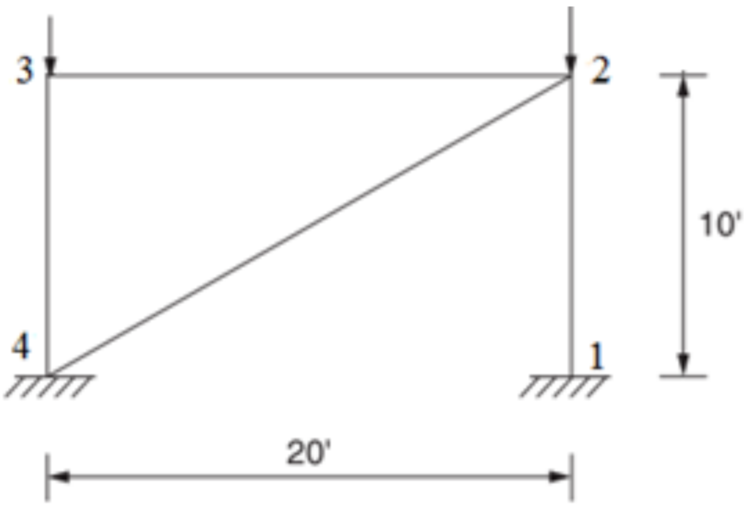


Fig. 5 - Details for Solved Exercise 3

At joint 2

$G_2 = \Sigma(E_cI_c/L_c)/\Sigma(E_gI_g/L_g)$

$= \frac{\frac{1}{10}}{\frac{2}{20}} = 1$

From the alignment chart for braced frames, the effective length factor is given as

$K_{21} = 0.75$- shown on the alignment chart with red line shown in figure 7.

Alignment Chart for Sway Frame

The alignment chart for sway frames is illustrated in Fig. 4. For a sway frame with rigid joints, the girders are assumed bent in double curvature and the alignment chart is based on a stiffness value for the girders of 6EI/L. If the far end of a girder is pinned, its stiffness is 3EI/L and the calculated (E_gI_g/L_g) value is multiplied by 0.5 before determining the value of $\Sigma(E_gI_g/L_g)$ and entering the chart. If the far end of a girder is fixed, its stiffness is 4EI/L and the calculated (E_gI_g/L_g) value is multiplied by 0.67 before determining the value of $\Sigma(E_gI_g/L_g)$ and entering the chart. In addition, the girder length is modified by AISC 360 Eq. (C-A-7-4) [1] to give a revised girder length of

$$L_g' = L_g(2 - M_F/M_N) \tag{3}$$

where ; M_F = far end girder moment and N = near end girder moment

M_F/M_N = +ve when the girder is in reverse curvature

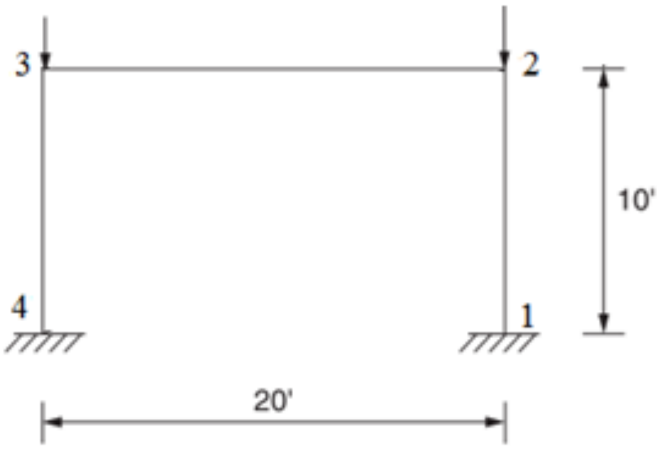


Fig. 6 - Details for Solved Exercise 4

Solved Exercise 4.Sway Frame Effective Length Factors by Alignment Chart

For the sway frame shown in Fig. 6, determine the effective length factor of column 21. The columns are adequately braced in the transverse direction.

For the fixed connection at joint 1, AISC 360 Commentary Sec. A-7.2 recommends a practical value of $G_1 = 1.0$.

At joint 2, because of the skew symmetrical loading and symmetrical structure the girder is in reverse curvature with $M_{32} = M_{23}$ and

$$L_g' = L_g(2 - M_{32}/M_{23}) = L_g(2 - 1) = L_g$$

$$G_2 = \Sigma(E_cI_c/L_c)/\Sigma(E_gI_g/L_g) = \frac{\frac{1}{10}}{\frac{2}{20}} = 1.0$$

From the alignment chart for sway frames, the effective length factor is K_{21} = 1.32- shown on the alignment chart with a green line shown in figure 7.

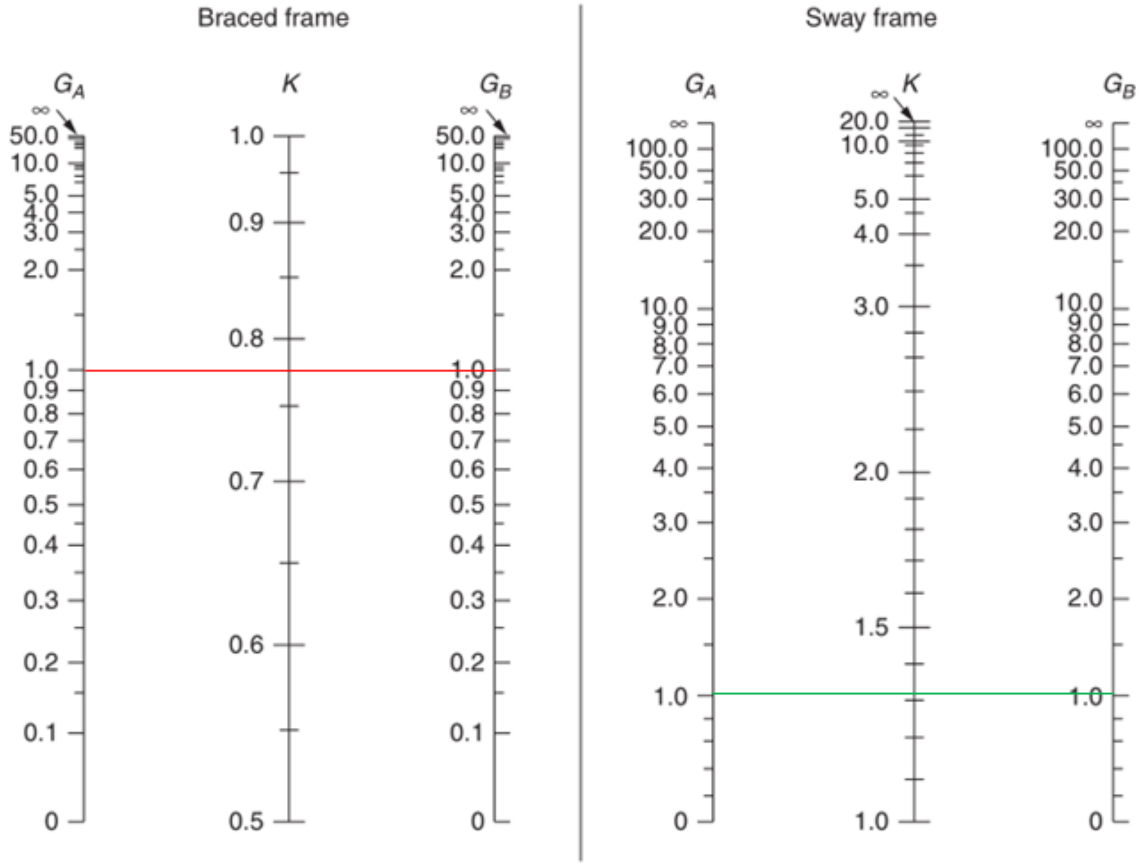


Fig. 7 - Alignment chart for effective length with result indications

LUI'S METHOD

A simple and straightforward method for determining the effective length factors for framed columns without the use the above explained methods was proposed by Lui [4]. Lui's formula takes into account both the member instability and frame instability effects explicitly. The K factor for the i-th column in a story was obtained in a simple form

$$\mathbf{K_i} = \sqrt{[(\frac{\pi^2 EI_i}{N_i^* L_i^2}) \times \Sigma(\frac{N^*}{L}) \times (\frac{1}{5\eta} + \frac{\Delta_1}{\Sigma H})]} \qquad (4)$$

N^*/L - sum of the factored axial force to length ratios of all members in a story,

$\Sigma\eta$ - sum of the member stiffness indexes of all members in the story.

$$\boldsymbol{\eta = [3 + 4.8\beta_m + 4.2\beta_m^2] \times EI/L^3} \qquad (5)$$

The member stiffness index η is illustrated in equation 5.

In equation (5) $\beta_m = M_A/M_B$ is the ratio of the smaller to larger end moments of the member, taken as positive if the member bends in reverse curvature and negative if the member bends in single curvature. Values for M_A and M_B are to be obtained from a first-order analysis of the frame subjected to a set of fictitious lateral forces applied at each story in proportion to the story factored gravity loads. Δ_1 in Eq. (4) is the inter-story deflection produced by these fictitious lateral forces, and Σ_H is the sum of the fictitious lateral forces at and above the story under consideration.

In the event that both M_A and M_B are zero, as in the case for a pinned-pinned leaner column, the ratio (θ_A/θ_B), where θ is the member end rotation with respect to its chord, should be used in place of (M_A/M_B) when calculating β_m .For instance, if the leaner column buckles in reverse curvature, η should be taken as $12EI/L^3$ (i.e. Taking $\beta_m = 1$), but if the leaner column bends in single curvature, η should be taken as $2.4EI / L^3$ (i.e. Taking $\beta_m = -1$). Detailed examples on this method is seen in the reference [pages 92-96 of [3]].

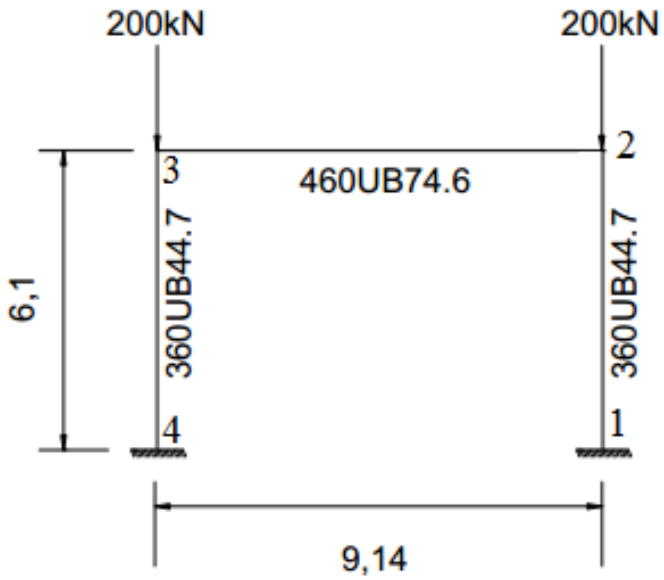


Fig. 8 - Frame with Columns

Solved Exercise 5. Determine the effective length factors for the rigid jointed frame shown figure 8

Solution

Using Fig.4.6.3.3 (b) of AS 4100 [3]:

Column 12

γ_1= N0./∞ = 0 → γ_1 = 1 (fixed support)

$\gamma_2 = \Sigma(EI / L)$ column / $\Sigma\, \beta_e(I /L)$ beam

$$\rightarrow\left[\frac{\left(\frac{554\times10^6}{10\times10^3}\right)}{\left(\frac{1\times986\times10^6}{20\times10^3}\right)}\right] = 1.12$$

Using Figure 4.6.3.3 (b) of AS 4100 [3] we obtain k_e = 1.35

LEMESSURIER METHOD

Considering that all columns in a story buckle simultaneously and strong columns will brace weak columns (Figure 9), a more accurate approach to calculate K-factors for columns in a sidesway frame was developed by LeMessurier[7]. The K_i value for the i-th column in a story can be obtained by the following expression:

$$\mathbf{K_i} = \sqrt{\frac{\pi^2 EI_i}{L_i^2 P_i}\left(\frac{\Sigma P + \Sigma C_L P}{\Sigma PL}\right)} \quad (6)$$

where P_i is the axial compressive force for member i, subscript i represents the i-th column, and ΣP isthe sum ofthe axial force ofall columnsin a story [9].

$$\mathbf{P_L} = \frac{\beta EI}{L^2} \quad (7)$$

$$\boldsymbol{\beta} = \frac{6(G_A+G_B)+36}{2(G_A+G_B)+G_AG_B+3} \quad (8)$$

$$\mathbf{C_L} = \left(\beta\frac{K_o^2}{\pi^2} - 1\right) \quad (9)$$

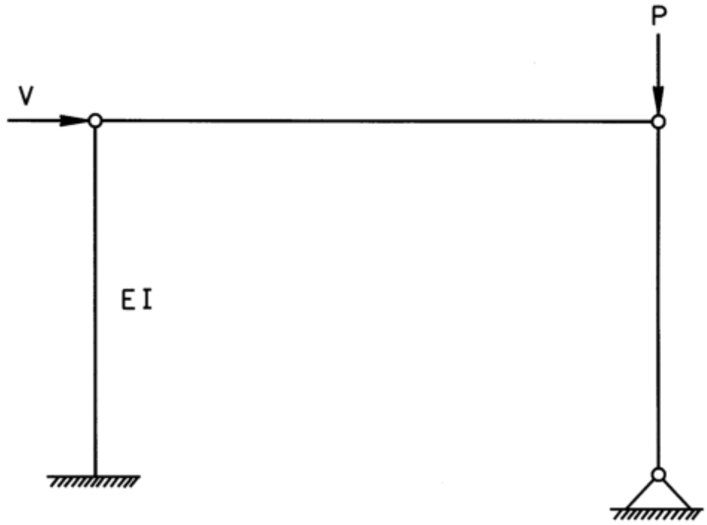


Fig. 9 - Sub-assemblage of the LeMessurier method

in which K_o is the effective length factor obtained by the alignment chart for unbraced frames, and P_L is only for rigid columns which provide side-sway stiffness. Duan, L.; Chen, W.F.[6] gives examples on this method.

Solved Exercise 6. A sway frame with columns of same height is shown in Figure 10. Determine elastic K-factors for columns by using the LeMessurier method. Member properties are:

Member	A in.^{2}	(mm^2)	*I* in.^{4}	(mm^4 x 10^8)	L in.	(mm)
AB	7.65	(4.935)	310	(1.29)	120	(3.048)
BD	21.5	(13.871)	620	(2.58)	240	(6.096)
CD	7.65	(4.935)	310	(1.29)	120	(3.048)

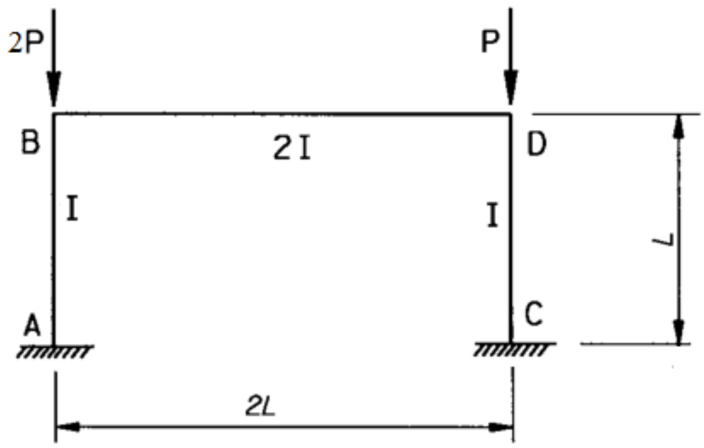


Fig. 10 - Frame dimensions and loads

Using Equation 6, we obtain;

$$K_{AB} = \sqrt{\frac{\pi^2 EI_{AB}}{L_{AB}^2 P_{AB}} \left(\frac{\Sigma P + \Sigma C_L P}{\Sigma PL}\right)}$$

$$\rightarrow K_{AB} = \sqrt{\frac{\pi^2 E(310)}{(120)^2 (P)} \left(\frac{3P + 0.495P}{0.271\text{E}}\right)} = 1.66$$

Conclusion

Using different methods to investigate the elastic length of steel frames, the effective length factors varies with a little difference which should not be paid much attention to. Future investigation effective length of steel frame columns will be done using Java codes for the analysis.

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